FE and IS

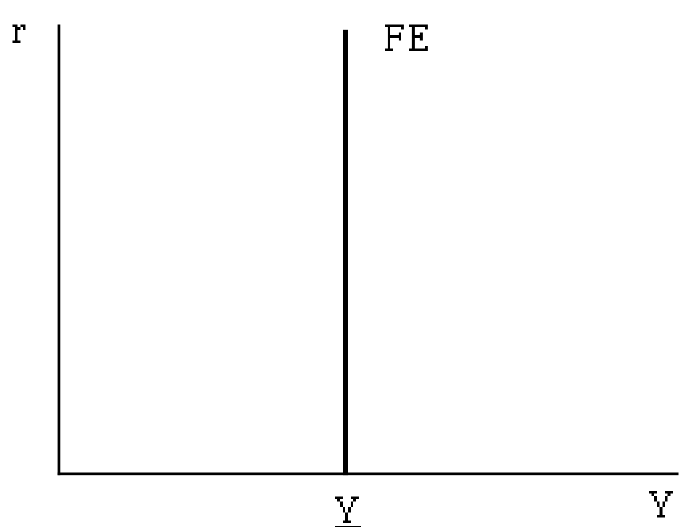
Chapter 3 gave us the relationship between output and employment in the long run. We assumed the following production for our economy

$$\overline{Y}=AF\left(\overline{K},\overline{N}\right)$$

$\overline{Y}$ = Full-employment output: amount of output produced when employment is at its full-employment level given the current level of the capital stock and the production functionn

$\overline{N}$ = full-employment level of employment

We want to relate the real interest rate and output. Recall, that in the goods market equilibrium in chapter 4 the endogenous variables were the real interest rate and desired savings and desired investment. We shall now diagram the relationship between the real interest rate and the full-employment level of output.

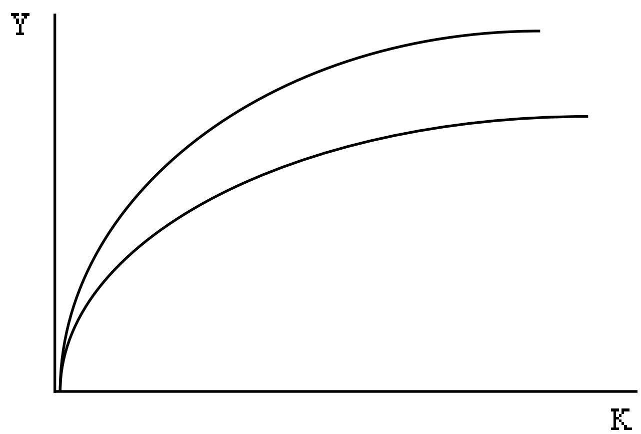


Why is it vertical?

ANS: When the labor market is in equilibrium, Y = Y no matter what the real interest rate! Now, you might being saying to yourself, "Come on man. Have you lost your senses? Certainly r must affect capital accumulation." Yes, it is true that r will affect the rate of capital accumulation by affecting investment, just as we learned in chapter 4. However, recall also from chapter 4 that the capital stock changes through changes in investment but with a lag. Therefore, changes in r today will affect investment that will affect tomorrow's capital stock. However, changes in r today will not affect today's capital stock.

Factors that shift the FE line:

Beneficial Supply Shock



Before the beneficial shock:

$$K_o \& N_o \Rightarrow Y_o$$

$$K_o \& N_o \Rightarrow Y_o = A_o F(K_o, N_o)$$

After the beneficial shock:

$$K_o \& N_o \Rightarrow A_1 F(K_o, N_o) = Y_1 > Y_o = A_o F(K_o, N_o)$$

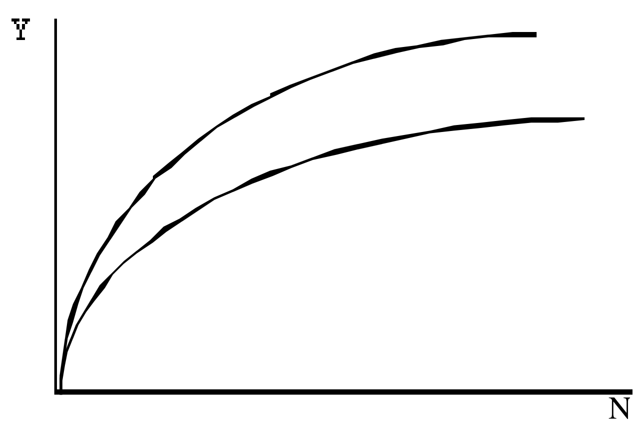
Where $A_1 > A_0$.

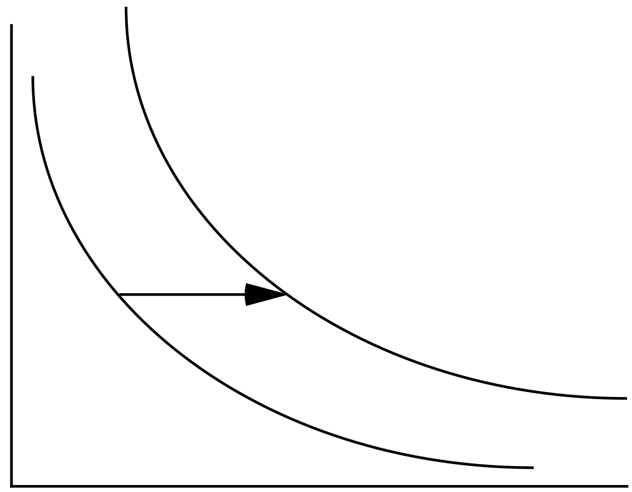
$$\frac{\partial Y}{\partial N} = AF_N = MPN$$

$$Y = AK^{\gamma}N^{(1-\gamma)} \quad \text{where}\ \gamma \in \lfloor 0, 1 \rfloor$$

$$\frac{\partial Y}{\partial N} = (1-\gamma)A\left(\frac{K}{N}\right)^{\gamma}$$

$$\text{If } \uparrow A \Rightarrow \uparrow\frac{\partial Y}{\partial N} = (1-\gamma)\uparrow A\left(\frac{K}{N}\right)^{\gamma}$$





Increase in labor supply:

$$\uparrow\overline{N} \Rightarrow \ \uparrow\overline{Y}=AF\left(\overline{K},\uparrow\overline{N}\right)$$

Increase in capital stock

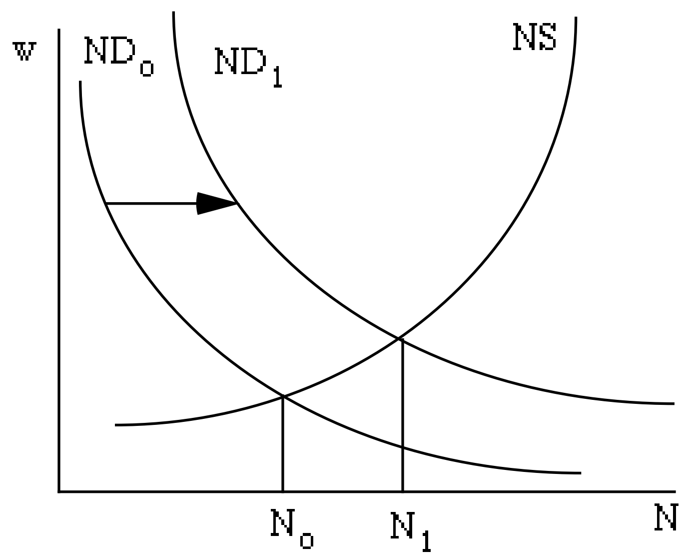
$$\uparrow K \Rightarrow \ \uparrow\overline{Y}=AF\left(\uparrow\overline{K},\overline{N}\right)$$

$$Y= AK^{\gamma}N^{(1-\gamma)} \ \text{ where } \gamma \in [0,1]$$

$$\uparrow\frac{\partial Y}{\partial N}= (1-\gamma)A\left(\frac{\uparrow K}{N}\right)^{\gamma}$$

$$\uparrow\frac{\partial Y}{\partial N}= (1-\gamma)A\left(\frac{\uparrow K}{N}\right)^{\gamma} \Rightarrow \uparrow ND \Rightarrow \text{labor demand shifts rightward}$$

$$\Rightarrow \ \uparrow\overline{N}$$



The IS Curve:

Goods Market Equilibrium:

$$Y = C^d + I^d + G$$

$\Rightarrow$ aggregate quantity of goods supplied
equals aggregate quantity of goods demanded

Rearranging the goods market equilibrium condition yields,

$$Y - C^d - G = I^d$$

Recalling the national saving relation

$$S^d = Y - C^d - G$$

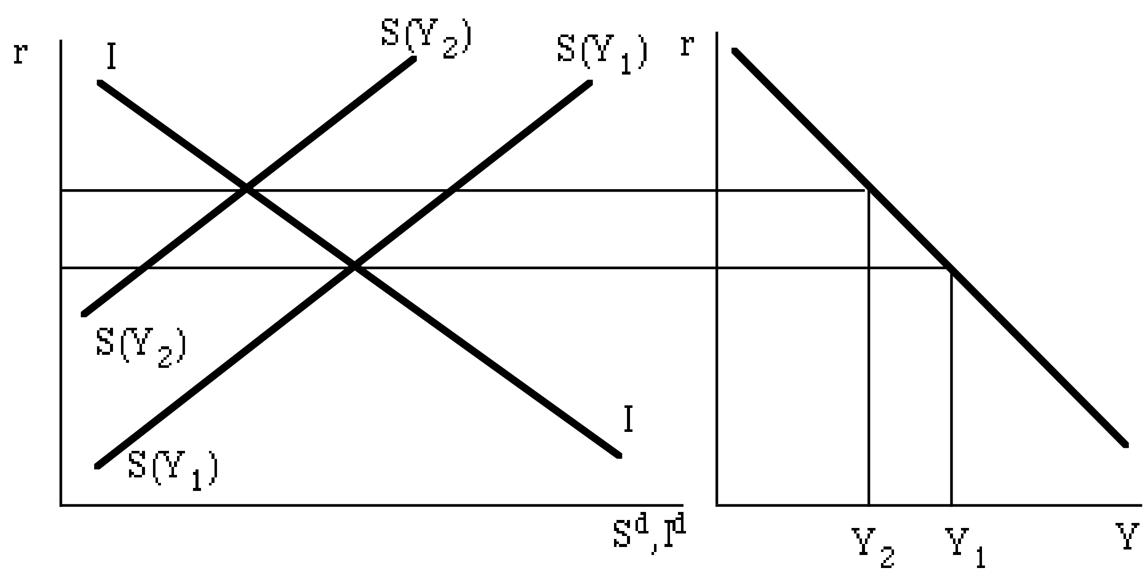
$$\Rightarrow S^d = I^d$$

We want to show the relationship between the interest rate and the level of output when the goods market is in equilibrium. Therefore, we must start with the goods equilibrium condition shown above. We ask the following question: "What are the various combinations of r and Y such that the goods market remains in equilibrium?" That is, as the interest rate changes, what must the level of output be so that the goods market is in equilibrium? Alternatively, we may ask, as the level of output changes what must the interest rate be so that the goods market is in equilibrium?

What happens as Y changes? Consider a fall in Y,

$$\downarrow Y \Rightarrow \downarrow C^d \Rightarrow \downarrow S^d = \downarrow Y - \downarrow C^d - \overline{G}$$

$$\Rightarrow \text{savings shifts leftward}$$



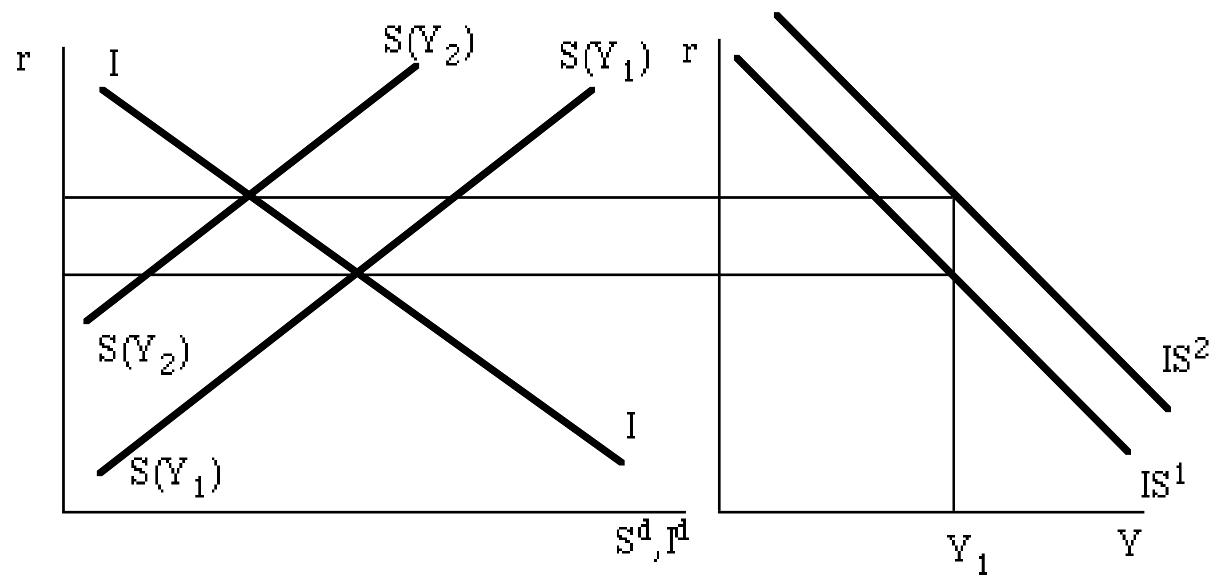
Notice that investment does not shift!! Why? Because no variables exogenous to desired investment have changed.

Exogenous variables for IS Curve

1. Expected future output increases
2.

$$\uparrow y_f^e \Rightarrow \uparrow C^d \Rightarrow \downarrow S^d \Rightarrow \text{leftward shift of saving curve}$$

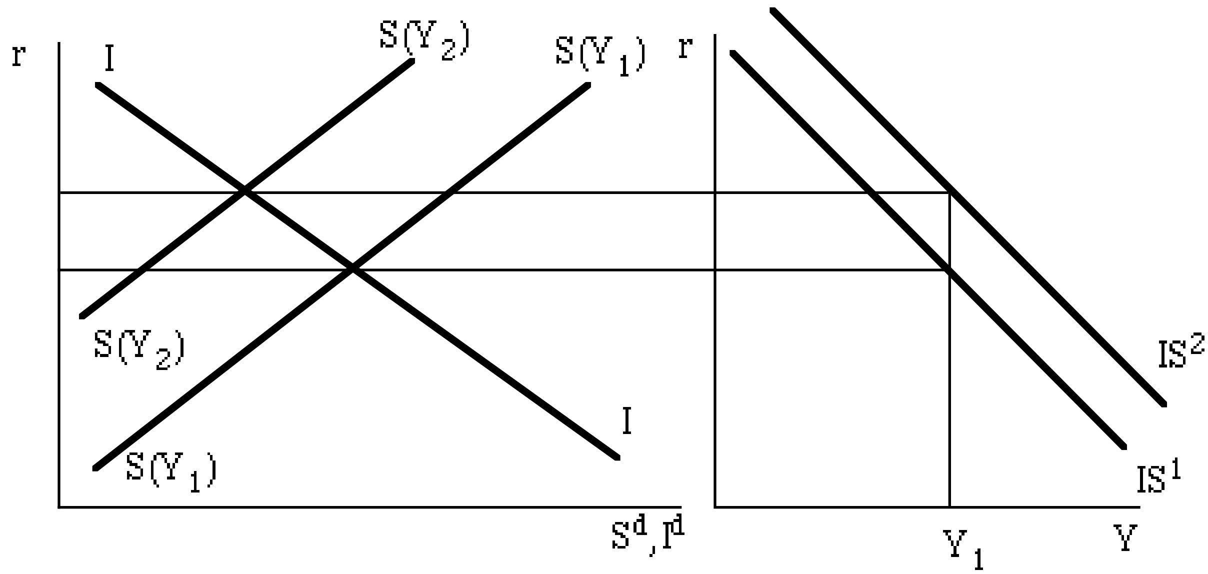
$$\Rightarrow \uparrow r \Rightarrow \text{real interest rate that clears the goods market is higher for any level of output}$$



2. Wealth

$$\uparrow y_f^e \Rightarrow \uparrow C^d \Rightarrow \downarrow S^d \Rightarrow \text{leftward shift of saving curve}$$

Real interest rate that clears the goods market is higher for any level of output



3. Government Purchases: We are assuming that there are no changes in taxes.

$$\uparrow G$$

$$\Rightarrow \left(\bar{Y}-\bar{T}\right) \Rightarrow \bar{y}_d \Rightarrow \bar{C}^d$$

$$\Rightarrow \downarrow S^d = \bar{Y} - \bar{C}^d - \uparrow G \Rightarrow \uparrow r \Rightarrow \text{IS shifts up}$$

4. Taxes: The effects of changes in taxes will depend on whether or not Ricardian equivalence holds.

Case 1: Assume that Ricardian equivalence does not hold

$$\uparrow T$$

$$\Rightarrow \downarrow\left(\bar{Y}-\uparrow T\right) \Rightarrow \downarrow \bar{y}_d \Rightarrow \downarrow \bar{C}^d$$

$$\Rightarrow \uparrow S^d = \bar{Y} - \downarrow C^d - \bar{G}$$

$$\Rightarrow \downarrow r \Rightarrow \text{IS shifts down}$$

case 2: Assume that Ricardian equivalence holds

$$\uparrow T \Rightarrow \downarrow\left(\overline{Y} - \uparrow T\right) \Rightarrow \downarrow y_d \Rightarrow \downarrow C^d$$

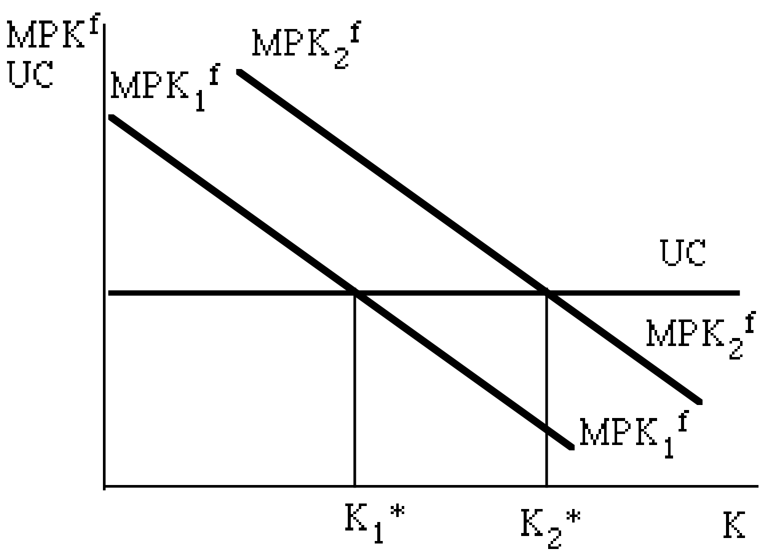
&

$$\downarrow T^f \Rightarrow \uparrow y_f^e \Rightarrow \uparrow C^d$$

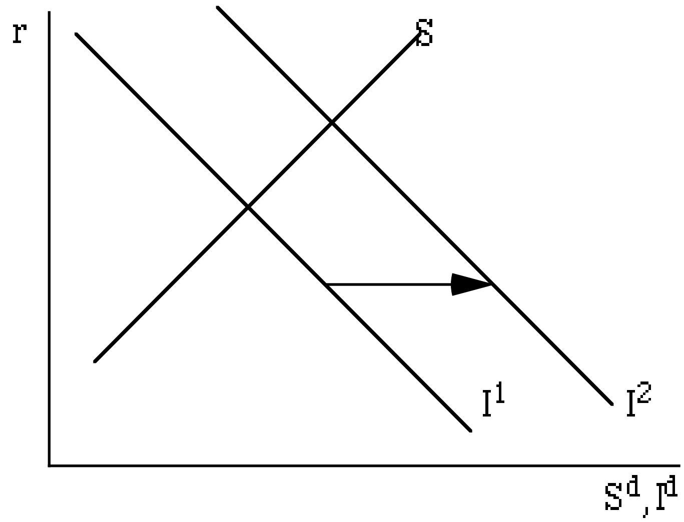
$$\Rightarrow \overline{S}^d = \overline{Y} - \overline{C}^d - \overline{G}$$

$$\Rightarrow \bar{r} \Rightarrow \overline{IS}$$

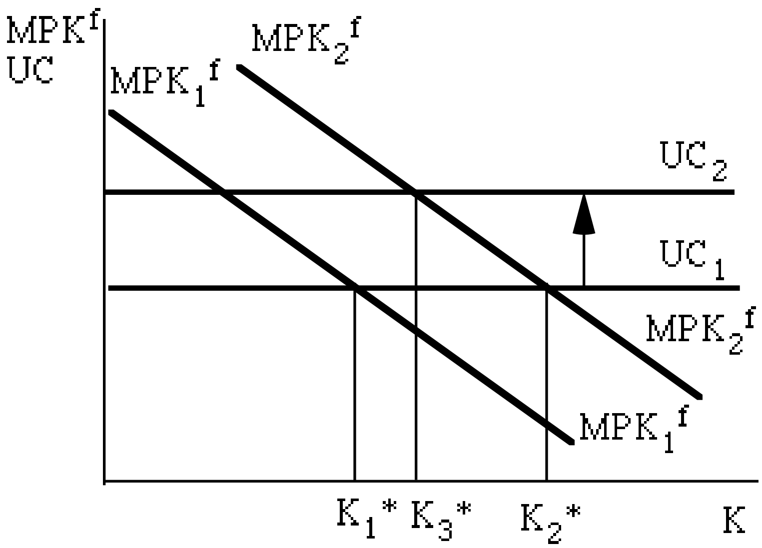
5. Expected future marginal product of capital:



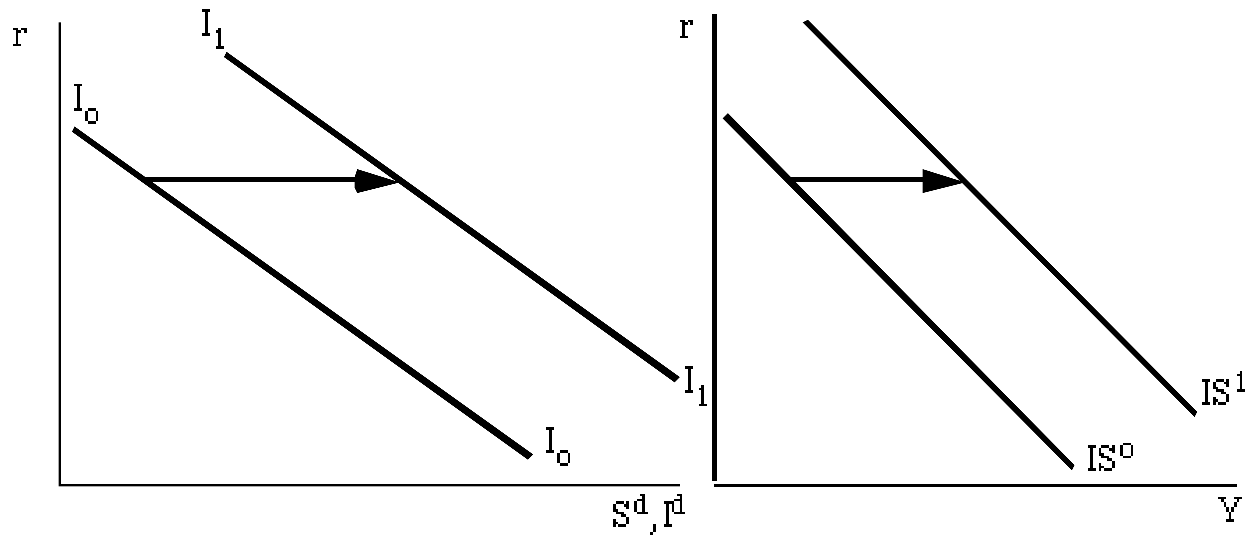
$$\uparrow MPK^f \Rightarrow \text{investment curve shifts rightward}$$



$$\uparrow r \Rightarrow \uparrow uc = (\uparrow r + d)p_k$$

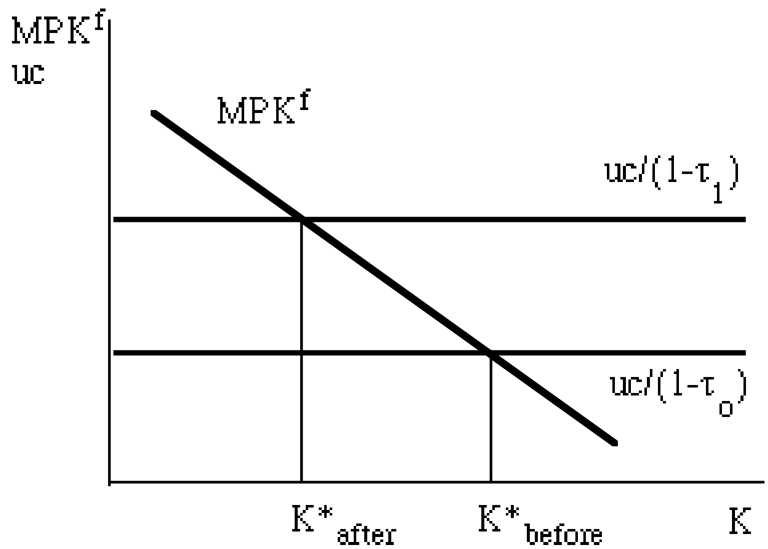


$\Rightarrow \uparrow r \Rightarrow$ real interest rate that clears the goods market is higher for any level of output



6. Effective tax rate on capital: consider the following case

$$\text{where } \tau_o < \tau_1$$



As the effective tax rate ($\tau \in (0,1)$) increases, the tax-adjusted user cost of capital increases. The desired capital stock is now less than it was before the tax was imposed.

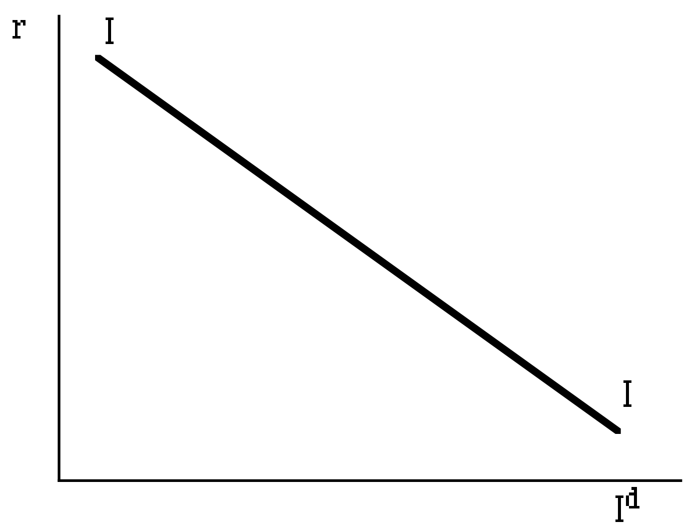
$$\Rightarrow \downarrow I_t = \downarrow K^* - K_t + dK_t$$

$$\uparrow\tau \Rightarrow \uparrow\left[\frac{uc}{\downarrow(1-\uparrow\tau)}\right] \Rightarrow \text{user cost shifts up}$$

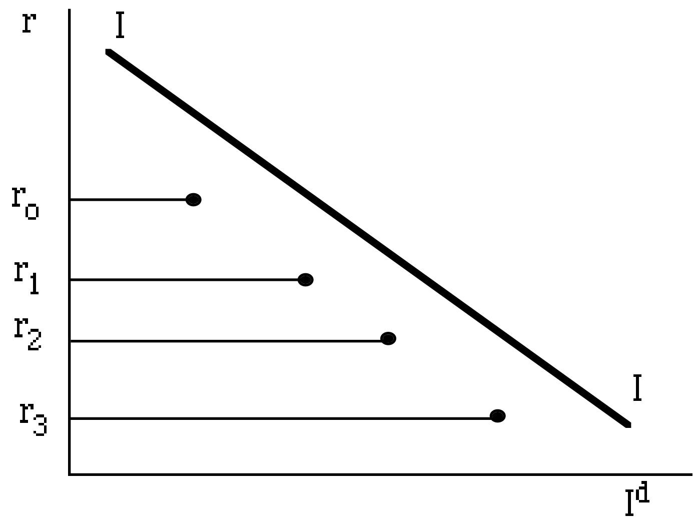
Recall the investment function

$$I^d\left(r; \overline{MPK}^f, \bar{\tau}\right)$$

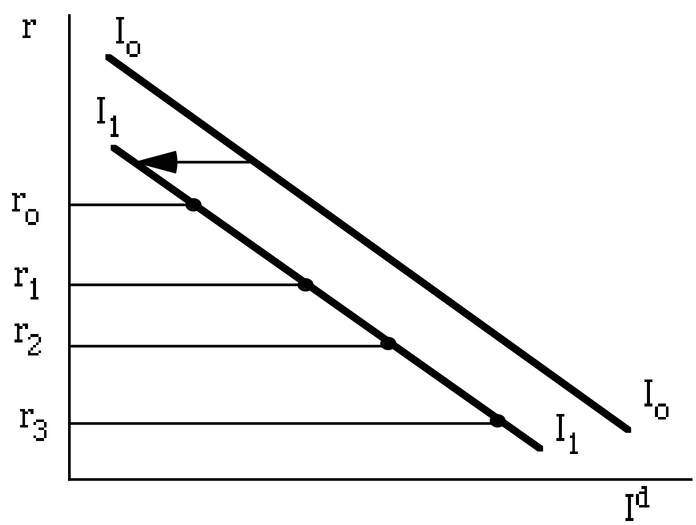
which is graphed as follows (please fill in)



When the effective tax rate changes, for a given interest rate, the level of desired investment changes. Note that this works for whatever the interest rate happens to be. So we can pick out an interest rate, run a line over to the desired investment schedule, and then ask, “Has the level of desired investment increased, decreased, or stayed the same when the effective tax rate increased?” From our discussion above, we demonstrated that for every rate of interest the level of desired investment has fallen. The only way that can be possible is if the desired investment schedule has shifted to the left.



What does this mean? It means that the investment schedule has shifted to the left. Now we can fill in the desired investment schedule by continuing this thought experiment for every single rate of interest. Now connect all of the dots and voila!! We have a new desired investment schedule.



This thought experiment can be conducted for any kind of shift that we may face, no matter what the curve represents. Always ask yourself the question, “For a given value of one of the endogenous variables, what has happened to the value of the other variable, when there is a change in an exogenous variable?”

Another way to write this is

$$\text{let } r = r_o \Rightarrow I^d\left(r_o; \overline{MPK}^f, \bar{\tau}\right) = I_o^d$$

*****Important*****: Here we denote the level of desired investment by I_o^d, this is the level of desired investment when the interest rate is r_o. $I_o^d(\cdot)$ denotes the function.

So,

$$\uparrow\tau \Rightarrow \downarrow I^d\left(r_o; \overline{MPK}^f, \uparrow\bar{\tau}\right) = \downarrow I_o^d$$

We can show this graphically utilizing the savings and investment diagram, which as you will recall gives us the real interest rate which brings the goods market into equilibrium. Please finish labeling the diagram below.