Lecture Notes #4 Ch. 4 Handout Tuesday 2/10/04

Recall the definition of national savings a closed economy

$$S = Y - C - G$$

Substitute in desired consumption for consumption

$$S^d = Y - C^d - G$$

Consider the following policy experiment: increased government expenditures financed by higher current taxes.

$$\uparrow G \,\&\, \uparrow T$$

What happens to disposable income, desired consumption and desired national savings?

$$\downarrow\left(\overline{Y} - \uparrow T\right) \Rightarrow \downarrow y_d \Rightarrow \downarrow C^d$$

$$\downarrow S^d = Y - \downarrow C^d - \uparrow G$$

Consider the following policy experiment: lump-sum tax cut today, financed by higher future taxes.

$$\overline{Y}, \overline{G}$$

What happens? **It Depends!!**
Scenario #1: If consumption depends entirely on disposable income

$$\downarrow S^d = \overline{Y} - \uparrow C^d - \overline{G}$$

$$\uparrow\left(\overline{Y}-\downarrow T\right)\Rightarrow\uparrow y_d\Rightarrow\uparrow C^d$$

Scenario #2: Consumption depends both on disposable income and future income and Ricardian equivalence holds.

$$\downarrow y_f,\uparrow y_d \,\&\, \Delta y_f=\Delta y_d\Rightarrow\overline{C}^d$$

$$\overline{S}^d=\overline{Y}-\overline{C}^d-\overline{G}$$

Recall that Ricardian equivalence refers to the notion that the government can either finance its current level of spending via taxes or by government borrowing. It does not matter which. Either route(assuming that Ricardian equivalence holds) will have precisely the same effects on the economy. Recall that if the government lowers taxes today, but does not changed current expenditure then the government must be financing its level of spending by borrowing. Consider the example in the book. Taxes fall by 10b. and current borrowing rises by 10b. This extra 10b. of government debt must be repaid in the future with interest. Therefore future income will be lower. Let's calculate the future value of this extra borrowing.

$$\text{present value }(1+r)\;=\;\text{future value}$$
$$\Rightarrow pv(1+r)=fv$$

$$pv=10b \;\&\; r\;=\;10\%$$

$$\Rightarrow fv=10b(1+10\%)$$

$$\Rightarrow fv=11b$$

So the $10b cut in taxes today means that the government must increase taxes by $11b in the future. Note that in our simple example we assumed that the government would raise the taxes the next year. Obviously, we can extend our analysis to incorporate any length of time. What if the government would increase taxes 3 years from today? We would use the following present value formula.

$$pv(1+r)^3=fv$$

What if the government would raise taxes n years from now? We would use the following formula.

$$pv(1+r)^n = fv$$

$$\Rightarrow pv = \frac{fv}{(1+r)^n}$$

Ricardian equivalence highlights the fact that in the long run all government purchases must be paid for by taxes. Therefore a cut in taxes today simply changes the timing of tax collections. The $10b increase in current income is equal to the $11b increase in taxes due a year later. To finance the same level of expenditures with a lower level of tax revenues, the government borrows money by selling bonds. So in our example the government will sell $10b worth of government bonds at an interest rate of r. A tax cut today which is financed by deficit spending will not make consumers better off. For every dollar that a person receives in tax cuts today they will have to pay a dollar plus interest one year from now. So, if they take the dollar and buy a government bond and receive the rate r, one year from now they will have exactly the amount needed to pay taxes.

Thus far we have explained the connection between desired consumption and desired savings. We have shown how fiscal policy affects desired savings. But, we need to explain the relationship between desired investment and the capital stock. Recall that in Chapter 3 we showed how the profit maximizing decision of the firm determined how many workers to hire. When we aggregated over all firms we were able to derive the aggregate labor demand schedule. Once again we shall look to the profit maximizing decision of the firm. But now we shall see how the firm determines how much capital to employ in the production process. This will enable us to derive the future marginal product of capital schedule. The firm will compare the future marginal product of capital to the user cost of capital and determine the desired level of capital.

Kyle's Bakery

$$p_k = \text{ real price of capital goods } (\$100/\text{cubic foot})$$

$d =$ rate of capital depreciation $(\%10/\text{year})$

$r =$ expected real interest rate $(8\%/\text{year})$

Considers the purchase of a piece of capital, in this case an oven, which has two components

1. Depreciation cost: Oven produces less over time. So even if Kyle doesn't sell the oven he is suffering a loss because his asset (the oven) will be worth less. For every $100 spent on the oven he looses $10 over the period of a year.

2. Interest cost: Interest payment on a loan to purchase the oven or if he purchases it out of profits then he is foregoing the interest earned on an interest-bearing asset like a government bond. For every $100 spent on the oven he looses $8 in foregone interest.

True Economic Cost:

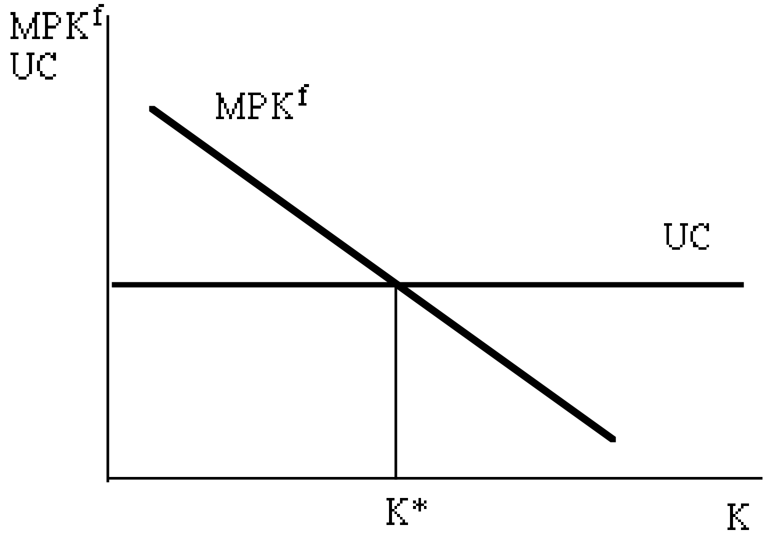
$$uc = rp_k + dp_k = (r + d)p_k$$

$$uc = \frac{\%8}{\text{year}} \frac{\$100}{\text{cubic foot}} + \frac{\%10}{\text{year}} \frac{\$100}{\text{cubic foot}} = \left(\frac{\%8}{\text{year}} + \frac{\%10}{\text{year}}\right) \frac{\$100}{\text{cubic foot}}$$

$$uc = \left(\frac{\%18}{\text{year}}\right) \frac{\$100}{\text{cubic foot}}$$

$$uc = \frac{\$18}{\text{year cubic foot}}$$

Therefore Kyle's user cost of capital is $18 per cubic foot per year.

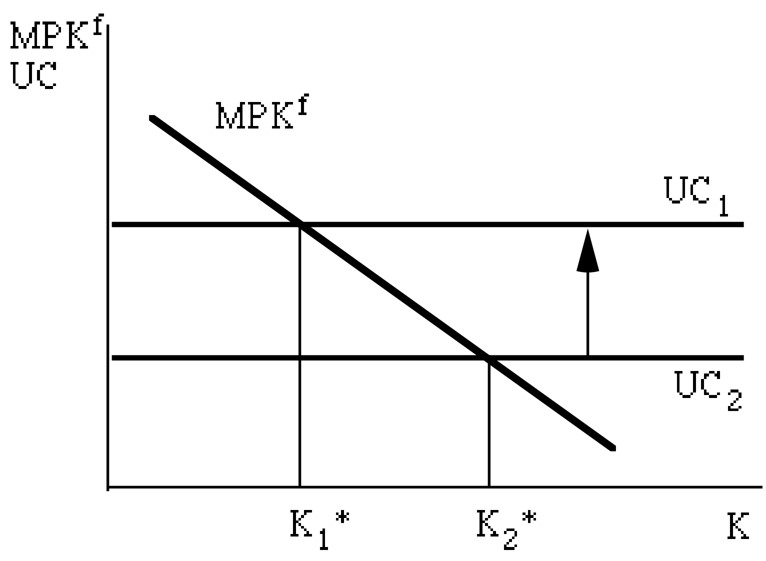


Changes that increase user cost of capital:

$$\uparrow r \Rightarrow \uparrow uc = (\uparrow r + d)p_k$$

$$\uparrow d \Rightarrow \uparrow uc = (r + \uparrow d)p_k$$

$$\uparrow p_k \Rightarrow \uparrow uc = (r + d)(\uparrow p_k)$$

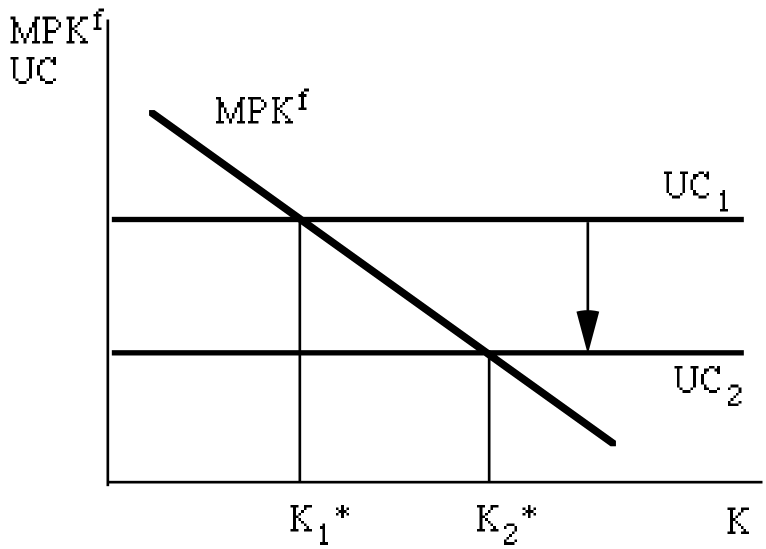


Changes that decrease the user cost of capital:

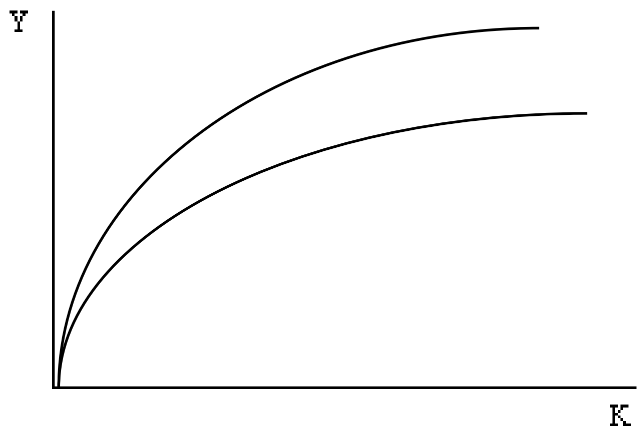
$$\downarrow r \Rightarrow \downarrow uc = (\downarrow r + d)p_k$$

$$\downarrow d \Rightarrow \downarrow uc = (r + \downarrow d)p_k$$

$$\downarrow p_k \Rightarrow \downarrow uc = (r + d)(\downarrow p_k)$$



Shifts in MPK: Anything that rotates the production function for the firm



Consider the firm's problem with revenue taxes

$$\underset{\{K,N\}}{\text{Max}}\,\Pi(K,N)$$

$$\Pi = (1-\tau)\overline{p}F(K,N) - \overline{W}\bullet N - \overline{UC}\bullet K$$

FONCS

$$(1-\tau)\overline{p}F_K - \overline{UC} = 0 \Rightarrow (1-\tau)\overline{p}F_K = \overline{UC} \Rightarrow (1-\tau)F_K = \overline{uc}$$

$$\Rightarrow (1-\tau)MPK^f = uc \Rightarrow MPK^f = \frac{uc}{(1-\tau)}$$

Tax-adjusted user cost of capital:

$$\frac{uc}{(1-\tau)}$$

Return to capital with revenue taxes:

$$(1-\tau)MPK$$

Relationship between capital stock and investment:

I_t =gross investment during year t

K_t = capital stock at the beginning of year t

K_{t+1} = capital stock at the beginning of year t+1

d = fraction of capital that depreciates each year

$$K_{t+1} - K_t = I_t - dK_t$$

$$\Rightarrow I_t = K_{t+1} - K_t + dK_t$$

Let

$$K^* = \text{desired capital stock}$$

Then

$$K^* = K_{t+1}$$

substitution yields

$$\Rightarrow I_t = K^* - K_t + dK_t$$

Goods Market Equilibrium:

$$Y = C^d + I^d + G$$

$\Rightarrow$ aggregate quantity of goods supplied
equals aggregate quantity of goods demanded

Rearranging the goods market equilibrium condition yields,

$$Y - C^d - G = I^d$$

Recalling the national saving relation

$$S^d = Y - C^d - G$$

$$\Rightarrow S^d = I^d$$

The goods equilibrium is not the same as the income expenditure identity.

$$Y = C + I + G \Leftrightarrow Y = C^d + I^d + G$$

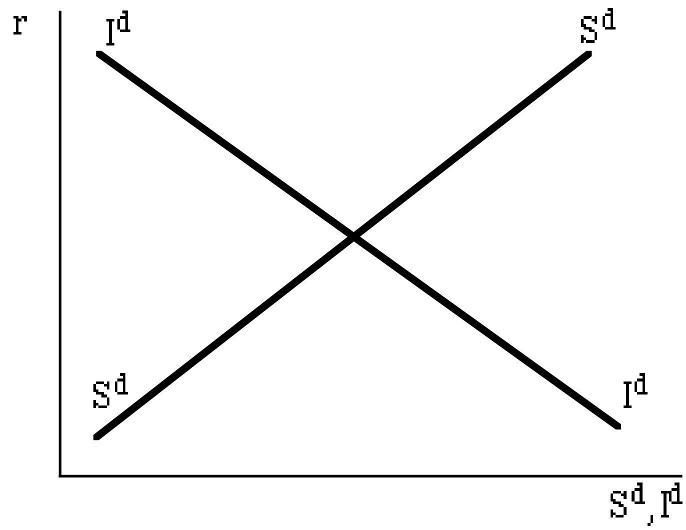
The lefthand equation is the income expenditure identity, where Y denotes actual income and C + I + G denotes actual spending. By definition this is always satisfied.

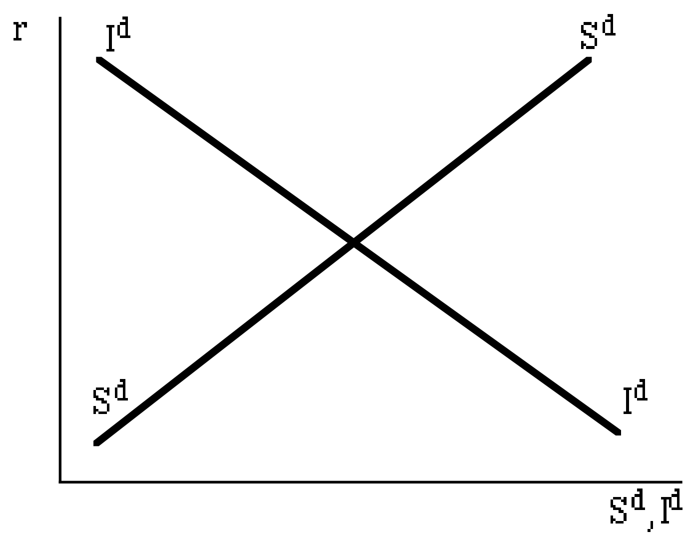
What if firms produce output faster than consumers want it and firms build up undesired inventories?

ANS: The income-Expenditure identity is still met. Undesired inventories are counted as part of total spending and are treated as Investment. But

$$Y > C^d + I^d + G$$

here desired spending is less than production. What are the forces that bring them into equilibrium? It is the adjustment of the interest rate.





A Quick and Dirty Guide to Savings and Investment

Why this guide? Even though we have been working with the national saving relation and the marginal conditions (MPK = uc and MPN = w) it is helpful to see a summary of the main points. Make sure that you understand the relationships between these marginal conditions and the goods equilibrium. So here we go . . . .

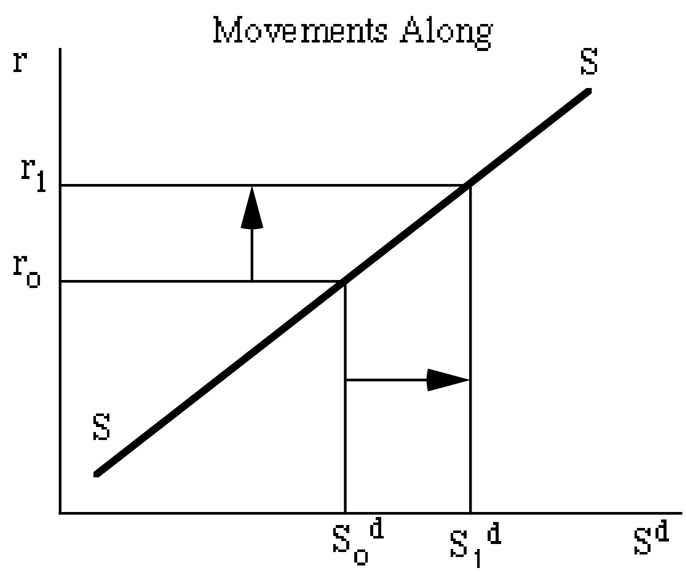
Savings Relation:
Exogenous variables: Y, expected future income, wealth, G, and taxes

Endogenous variables: r, S^d

Investment:
Exogenous variables: τ, anything changing MPK^f

Endogenous variables: r, I^d

Anytime we talk about shifts in the savings schedule we are talking about a change in one of the exogenous variables. When we talk about the relationship between r and S^d we are talking about movements along the savings schedule.



As a thought experiment consider the following: real interest rate rises, what happens to desired savings?

ANS: We want to find out what happens to desired national savings as the rate of interest goes up so use the desired national savings relation

$$\uparrow r \Rightarrow \uparrow S^d$$

It is useful to write the desired national savings function explicitly

$$S^d\left(r; \overline{y}_f^e, \overline{Y}, \overline{T}, \overline{T}_f^e, \overline{W}\right)$$

$r \equiv$ real interest rate

$\overline{y}_f^e \equiv$ expected future income

$\overline{Y} \equiv$ current income

$\overline{T} \equiv$ taxes

$\overline{T}_f^e \equiv$ expected future taxes

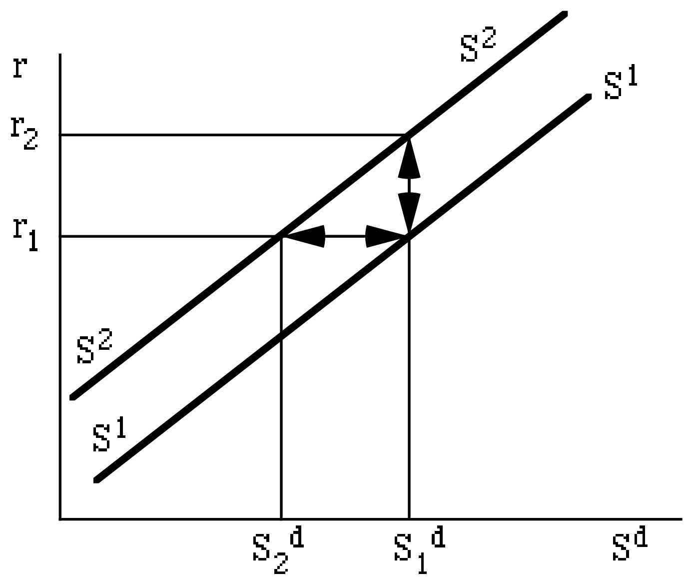
$\overline{W} \equiv$ wealth

The bar over the variable denotes the fact that it is an exogenous variable. These variables are not explained by the model. They are "given" from outside the model. The interest rate does not have a bar because it is an endogenous variable. When exogenous variables change the schedule will shift.

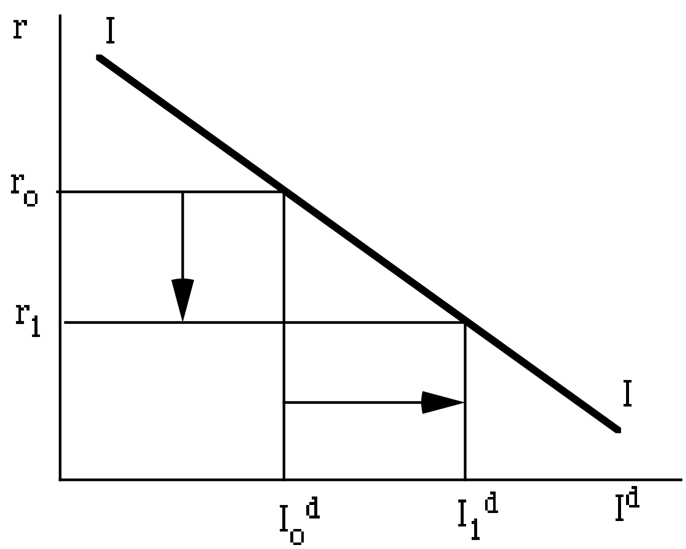
Consider the case of a shift in national savings. There are two ways that we can think about this shift.

1. The shift occurred because one of the exogenous variables changed. We can now consider what happens to desired national savings at the very same interest rate, r_1. After the shift, the level of desired savings is less than it was before at the same interest rate.

2. Alternatively, we can consider what happens to the interest rate at the very same quantity of desired national savings, S_1^d. After the shift, for the same level of savings to be desired the interest rate must rise.



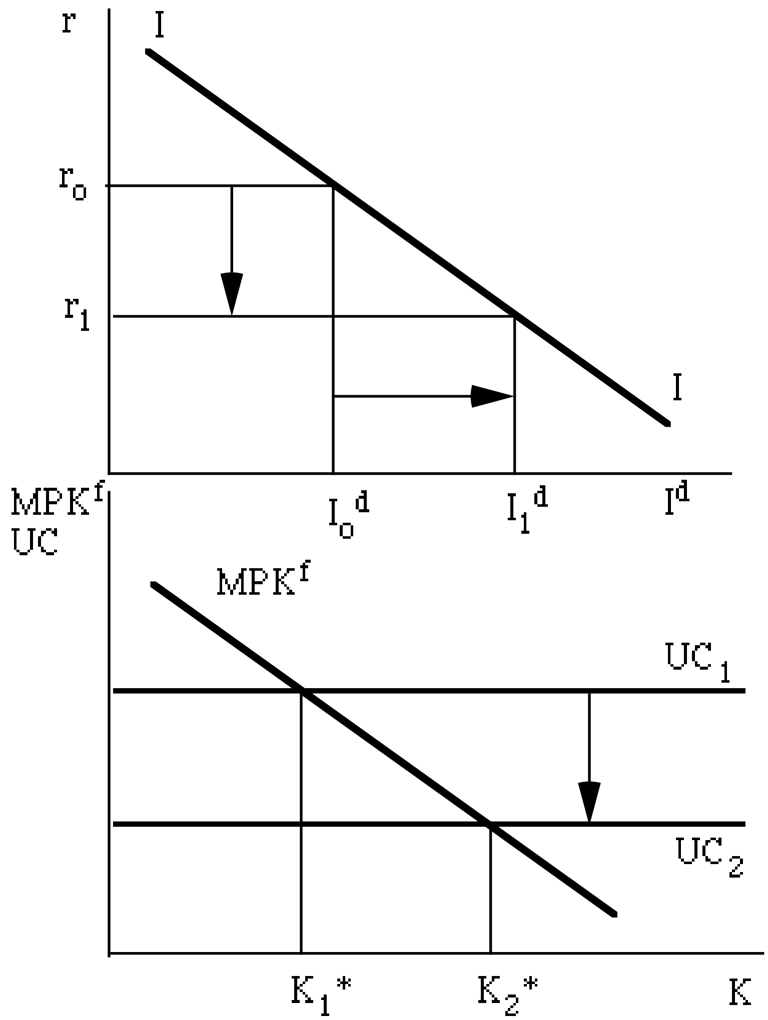
Consider the investment schedule. When r changes I^d. This is a movement along the investment schedule.



When there is an exogenous change in investment we refer to either of the two exogenous variables on which investment depends.

$\overline{MPK}^f \equiv$ future marginal productivity of capital

$\bar{\tau} \equiv$ effective taxe rate



The second diagram shows precisely what we mean when we argue that the investment schedule is a function of the future marginal productivity of capital. As the interest rate changes, we move along the investment schedule. A lower interest rate means that the user cost has fallen. A lower user cost represents a shift of the user cost curve, along a stable future marginal productivity of capital curve. Now it is certainly true that the magnitude of the marginal productivity of capital is changing as the interest rate changes but the function is not. What would cause the function to change? Anything that would rotate the firm's production function.

Now why does τ shift the investment schedule? Recall the firm's first order necessary condition regarding the purchase of capital with revenue taxes.

$$\underset{\{K,N\}}{\text{Max}}\,\Pi(K,N)$$

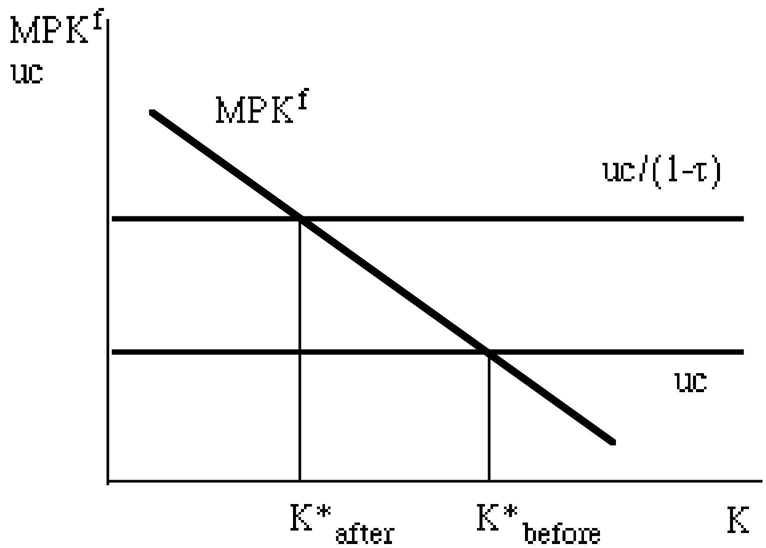
$$\Pi = (1-\tau)\overline{p}F(K,N) - \overline{W}\bullet N - \overline{UC}\bullet K$$

FONCS

$$(1-\tau)\overline{p}F_K - \overline{UC} = 0 \Rightarrow (1-\tau)\overline{p}F_K = \overline{UC} \Rightarrow (1-\tau)F_K = \overline{uc}$$

$$\Rightarrow (1-\tau)MPK^f = uc \Rightarrow MPK^f = \frac{uc}{(1-\tau)}$$

Let's diagram this.



As the effective tax rate ($\tau \in (0,1)$) increases, the tax-adjusted user cost of capital increases. The desired capital stock is now less than it was before the tax was imposed.

$$\Rightarrow \downarrow I_t = \downarrow K^* - K_t + dK_t$$